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PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

[Errata: *To the number of each problem for solution in the issue of February, 1919, 4 should be added*].

2757. Proposed by ERNEST P. LANE, Rice Institute, Houston, Texas.

Integrate by quadrature the differential equation

$$\frac{d^2y}{dx^2} - 3y\frac{dy}{dx} + y^3 = 0.$$

2758. Proposed by L. RICHARDSON, Vancouver, B. C., Canada.

Prove that, if r be a positive integer,

$$\int_0^{\pi/2} \frac{\sin (2r+1)\psi}{\sin \psi} d\psi = \frac{\pi}{2}$$

and

$$\int_0^{\pi/2} \frac{\sin 2r\psi}{\sin \psi} d\psi = 2\left\{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^{r-1}}{2r-1}\right\}.$$

2759. Proposed by J. L. RILEY, Stephenville, Texas.

Solve the simultaneous functional equations

$$\Phi(x+y) = \Phi(x) + \frac{\Phi(y)\cdot\psi(x)}{1 - \Phi(x)\Phi(y)},$$

$$\psi(x+y) = \frac{\psi(x)\psi(y)}{1 - \Phi(x)\Phi(y)}.$$

2760. Proposed by CHARLES N. SCHMALL, New York City.

In an arithmetical progression, if s_n be the sum of the first n terms, s_{2n} the sum of the first $2n$ terms, and s_{3n} the sum of the first $3n$ terms of the same series, prove that $s_{2n} - s_n = \frac{1}{3}s_{3n}$.

2761. Proposed by W. W. DENTON, Ann Arbor, Michigan.

Find the length of the side of an equilateral triangle whose vertices are at given distances a, b, c, from a given point.

SOLUTIONS OF PROBLEMS.

411 (Calculus) [June, 1916]. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

Prove that the volume bounded by the surface $f(x, y, z) = 0$ is $1/3\iint (z - x(\partial z/\partial x) - y(\partial z/\partial y))\,dxdy$ integrated over the area determined by projecting the surface on the xy-plane.

Solution by Elijah Swift, University of Vermont.

The above problem is not stated accurately for all cases. The integral should be taken over the entire surface,—that is we have a surface integral, where the element of surface, dS, is replaced by its projection on the xy-plane in each element of the sum. This is not the same as integrating over the area described above.

Let us assume that the above surface is a closed, finite one with no double points or singularities. Then the direction cosines of the normal at any point are proportional to $-\partial z/\partial x$, $-\partial z/\partial y$, 1. If we denote the direction cosines by α, β, γ, and remember that $\gamma\cdot dS\cdot = dxdy$, the given integral becomes $1/3\iint(\gamma z + \alpha x + \beta y)dS$ taken over the surface. By a special case of Green's Theorem (B. O. Peirce, *Short Table of Integrals*, formula 883) this becomes $1/3\iiint(1 + 1 + 1)dxdydz$, taken throughout the volume inclosed by the surface, which is clearly the volume required.

That the above formula does not give the volume bounded by $f(x, y, z) = 0$, the xy-plane and a cylinder whose elements are parallel to the z-axis, may be readily seen by applying it to the plane $z = c$, in which case it gives a result one-third as large as the correct result.

Also solved by the Proposer.

239 (Number Theory) [March, 1916]. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

Give a general method for determining the solution in integers of the equation

$$x^r - 10xy - (n + 1) + y = 0,$$

where n and r are positive integers.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Solving for y, $y = \dfrac{x^r - (n + 1)}{10x - 1}$, which must be an integer. Since the denominator is prime to 10 and the numerator integral, y will be an integer if $\dfrac{10^r x^r - (n + 1)\cdot 10^r}{10x - 1}$ is. Dividing algebraically the remainder is $1 - (n + 1)\cdot 10^r$. If then $\dfrac{10^r(n + 1) - 1}{10x - 1}$ is an integer, so is y. Hence the general process (perhaps not that desired) is the following: form $10^r(n + 1) - 1$ and factor it. Equate $10x - 1$ to any factor whose last digit is 9, and we have an integral solution.

261 (Number Theory) [March, 1917]. Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that for any positive integer n (excluding powers of 2) positive integers $a_1, a_2, a_3, \cdots, a_k$ which are less than $n/2$ can be chosen in such a way that

$$2^k \cos (a_1\pi/n) \cos (a_2\pi/n) \cos (a_3\pi/n) \cdots \cos (a_k\pi/n) = 1.$$

SOLUTION BY C. F. GUMMER, Queen's University.

Since n is not a power of 2, it is of the form $(2k + 1)l$. The equation

$$\cos (2k + 1)x - \cos (2k + 1)\alpha = 0$$

has $2k + 1$ distinct roots in $\cos x$, when $\cos (2k + 1)\alpha \neq 1$, the roots being

$$\cos \left(\alpha + \frac{2i\pi}{2k + 1}\right), \qquad i = 0, 1, \cdots, 2k.$$

Also $\cos (2k + 1)x = 2^{2k} \cos^{2k+1} x - \cdots$, the absolute term being zero. Hence,

$$2^{2k} \prod_{i=0}^{2k} \cos \left(\alpha + \frac{2i\pi}{2k + 1}\right) = \cos (2k + 1)\alpha.$$

By taking the limit of each side when $\alpha \rightarrow 0$,

$$2^{2k} \prod_{i=0}^{2k} \cos \frac{2i\pi}{2k + 1} = 1,$$

that is,

$$\left\{2^k \prod_{i=0}^{k} \cos \frac{2i\pi}{2k + 1}\right\}^2 = 1,$$

or

$$2^k \prod_{i=0}^{k} \cos \frac{2i\pi}{2k + 1} = \pm 1.$$

By taking $j = 2i$, when $i \leqq k/2$ and $j = 2k + 1 - 2i$ when $i > k/2$, we get

$$2^k \prod_{j=1}^{k} \cos \frac{j\pi}{2k + 1} = + 1,$$

which takes the required form if $a_j = jl$.